# OBSTRECLES WHICH STAND IN THE WAY OF STUDENTS LEARNING THE LIMIT CONCEPT .

Dr. Dheeraj Kumar.

P.G. Department of Mathematics , Patna University, Patna, Bihar.

Email: dheerajkr2011@gmail.com

**ABSTRACT**

The mathematical concept of a limit is a particularly difficult notion, typical of the kind of thought required in advanced mathematics. It holds a central position in mathematical analysis providing a foundation for the theory of approximation, or continuity, or the entire subject matter of differential and integral calculus. One of the greatest difficulties in teaching and learning the limit concept lies not only in its richness and complexity, but also the fact that its cognitive universe cannot be generated purely from the mathematical definition.

**KEYWORDS :** Limit, Conceptual obstacles, Spontaneous conceptions, Kinetic infinitists, Cognitive obstacle, Epistemological obstacles,

## INTRODUCTION

The distinction between the definition and the concept itself is didactically very important. It may be noted that the definition of a limit is one thing, while acquiring the fundamental conception is another thing. One facet is the idea of approximation, usually first encountered through a dynamic notion of limit, and the way in which the concept of limit is put to work to resolve real problems which rely not on the definition but on many diverse properties of the intuitive concept. Starting from such a point of view students often believe that they "understand" the definition of a limit without truly acquiring all the implications of the formal concept. Students are often able to complete many of the exercises they are asked to perform without having to understand the formalism of the definition at all. Meanwhile, the quantifiers "for all", "there exists", which occur in epsilon – delta definitions, have their own meanings in everyday language, subtly different from those encountered in the definition of the limit concept. From such beginning arise conceptual obstacles which may cause serious difficulties.

In teaching mathematics, certain aspects of the limit concept are given greater emphasis which are revealed by a review of the curriculum, the textbook, exercises and examinations. In the first half of the twentieth century, mathematics texts used the notion of Limit in an intuitive manner without a formal definition to introduce the definition of the derivative. Books generally devoted a chapter to the general limit concept including a formal definition, a statement of its uniqueness, and theorems about arithmetic operations applied to limits. The exercises, however, did not concentrate on the limit concept, but on inequalities, the notion of absolute value, the idea of a sufficient condition and, above all, on operations: the limit of a sum, of a product, and so on. These exercises are far more related to algebra and the routines of formal differentiation and integration than to analysis. Given such a bias in emphasis it is therefore little wonder that students pick up implicit belief about the way in which they are expected to operate. Different investigations which have been carried out show only too clearly that the majority of students do not master the idea of a limit, even at a more advanced stage of their studies. This does not prevent them working out exercises, solving problems and succeeding in their examinations!

In this paper we will study some didactic aspects of the idea of limits: concepts linked to this notion various obstacles which stand in the way of students learning the limit concept, and discuss various strategies for teaching the limit concept.

**[1] Spontaneous conception of limit and Mental model :**
For most mathematical concepts, teaching does not begin on virgin territory. In the case of limits, before any teaching on this subject the student already has a certain number of ideas, intuitions, images, knowledge, which come from daily experience, such as the colloquial meaning of the terms being used. We describe these conceptions of an idea, which occur prior to formal teaching, as *spontaneous conceptions*. When a student participates in a mathematics lesson, these ideas do not disappear – contrary to what may be imagined by most teachers. These spontaneous ideas mix with newly acquired knowledge, modified and adapted to form the students personal conceptions. We know that in order to resolve a problem, we do not in general call uniquely on adequate scientific theory, but on natural or spontaneous reasoning, which is founded on these spontaneous ideas. This phenomenon is well known in the empirical and theoretical development of scientific concepts since Bachelard in the nineteen-thirties, but it is only in the last decade that it has been fully realized that exactly the same forces operate in the apparent logic of mathematics.

"In the case of the limit concept, we observe that the words 'tends to' and 'limit' have a significance for the students before any lessons begin". In fact students continue to rely on these meanings after they have been given a formal definition. Investigations have revealed many different meanings for the expression `tends towards':

- To approach (eventually staying away from it)
- To approach ………. Without reaching it
- To approach …. Just reaching it
- To resemble (without any variation, such as "this red tends towards green")

The word limit itself can have many different meanings to different individuals at different times. Most often it is considered as an 'impassible limit', but it can also be:

- An impossible limit which is reachable,
- An impossible limit which is impossible to reach
- A point which one approaches, without reaching it,
- A point which one approaches and reaches
- A higher(lower) limit,
- A maximum or minimum
- An interval
- That which comes "immediately after' what can be attained
- A constraint, a ban, a rule,
- The end, the finish. (Cornu,1983)

According to situations from one student to another the meaning given to words varies; for one student also it may have many meanings. Spontaneous ideas live on a long time; investigations show that they may remain with students at a much more advanced stage of learning. In the face of a variety of spontaneous notions and the student's growing awareness of the formalisms it easily happens that contradictory ideas may be held simultaneously in the mind of an individual, leading to a global "concept image" which contains potential conflicting factors in the sense of Tall & Vinner (1981).

Aline Robert (1982a, b) has studied different models which students may hold of the notion of the limit of a sequence. Despite the fact that students have been given a formal definition of a sequence, when asked to describe the notion of a sequence, their descriptions are not precise and is often coloured by their previous experience or understanding. Some students suggested primitive, rudimentary models, reminiscent of those which might be evoked spontaneously, such as:

- Stationary: "The final terms always have the same value",
- Barrier: "The values cannot pass *l* ":

In addition there were more models which arose more from the formal teaching:

- Monotonic and dynamic – monotonic :
  "a convergent sequence is an increasing sequence bounded above".
  "a convergent sequence is an increasing sequence which approaches a limit".
- Dynamic :
  "$u_n$ tends to l";
  "$u_n$ approaches l";
  "the distance from $u_n$ to l becomes small";
  "the values approaches a number more and more closely".
- Static :
  "The $u_n$ are in an interval near l";
  "the $u_n$ are grouped round l";

"The elements of the sequence end up by being found in a neighbourhood of l":

- Mixed : a mixture of those above.

Once more she found these models influencing the manner in which students at university solved problems. There is clearly no single notion of limit in the minds of students. It is evident that the students have a variety of concept images.

Moreover, it is also evident that the initial teaching tends to emphasize the process of approaching a limit, rather than the concept of the limit itself. The concept imagery associated with this process, as exemplified above, contains many factors which conflict with the formal definition ("approaches but cannot reach", "cannot pass", "tends to"). Thus it is that students develop images of limits and infinity which relate to misconceptions concerning the process of "getting close" or "growing large" or "going on forever".

In an ethnographic study of the conceptions of student concerning limits and infinity, Sierpinska (1987) analysed the concept images of 31 sixteen year – old pre – calculus mathematics and physics students. She then classified them into groups which she labeled with a single name for each group:

*Micheal and Christopher are unconscious infinities (at least at the beginning): they say "infinite", but think "very big ".... For both of them the limit should be the last value of the term.... For Micheal this last value is either plus infinity (a very big positive number) or minus infinity.... It is not so for Christopher who is more receptive to the dynamic changes of values of the terms. The last value is not always tending to infinity; it may tend to some small and known number.*

*George is a conscious infinitist: infinity is about something metaphysical, difficult to grasp with precise definitions. If mathematics is to be an exact science then one should avoid speaking about infinity and speak about finite number only. In formulating general laws one can use letters denoting concrete but arbitrary finite numbers. In describing the behaviour of sequences the most important thing is to characterize the nth term by writing the general formula. For a given n one can then compute the exact value of the term or one can give an approximation of this value.*

*Paul and Robert are kinetic infinitists : the idea of infinity in them is connected with the idea of time. .... Paul is a potentialist : to think of some whole, a set or a sequence, one has to run in thought through every element of it. It is impossible to think this way of an infinite number of elements. The construction of an infinite set or sequence can never be completed. Infinity exists potentially only. Robert is a potential actualist: it is possible to make a "jump to infinity" in thought: the infinity can potentially be ultimately actualized. For both, Paul and Robert, the important thing is to see how the terms of the sequence change, if there is a tendency to approach some fixed value, for Paul, even if the terms of a sequence come closer and closer so as differ less than any given value they will never reach it. Robert thinks theoretically the terms will reach it in the infinity.*

In this way she exhibits timeless conflicts about limits and infinity which have been with us since time immemorial and which continue to hold in our students today

**[2] COGNITIVE OBSTACLES:**

The notion of a *cognitive obstacle* is interesting to study to help identify difficulties encountered by students in the learning process, and to determine more appropriate strategies for teaching. It is possible to distinguish several different types of obstacles: *genetic and psychological obstacles*[6] which occur as a result of the personal development of the student, *didactical obstacles* which occur because of the nature of the teaching and the teacher, and *epistemological obstacles* which occur because of the nature of the mathematical concepts themselves. In planning to teach a mathematical concept it is of the utmost importance to determine the possible obstacles, particularly the endemic epistemological obstacles.

The term was introduced by Gaston Bachelard (1938):

*"We must pose the problem of scientific knowledge in terms of obstacles. It is not just a question of considering external obstacles, like the complexity and the transience of scientific phenomena, nor to lament the feebleness of the human senses and spirit. It is in the act of gaining knowledge itself, to know intimately what appears, as an inevitable result of functional necessity, to retard the speed of learning and cause cognitive difficulties. It is here that we may find the causes of stagnation and even of regression, that we may perceive the reasons for the inertia, which we call epistemological obstacles."*

He goes on to say: "We encountered new knowledge which contradicts previous knowledge, and in doing so must destroy ill – formed previous ideas." He indicated that epistemological obstacles occur both in the historical development of scientific thought and in educational practice, for him, epistemological obstacles have two essential characteristics:

- They are unavoidable and essential constituents of knowledge to be acquired,
- They are found, at least in part, in the historical development of the concept.

Many authors have become interested in epistemological obstacles. Guy Brousseau defines an epistemological obstacle as "*knowledge which functions well in a certain domain of activity and therefore becomes well - established, but then fails to work satisfactorily in another context where it malfunctions and leads to contradictions".* It therefore becomes necessary to destroy the original insufficient, malformed knowledge, to replace it with new concept which operates satisfactorily in the new domain. The rejection and clarifying of such an obstacle is an essential part of the knowledge itself; the transformation cannot be performed without destabilizing the original ideas by placing them in a new context where they are clearly seen to fail. This therefore requires a great effort of cognitive reconstruction.

## [3] EPISTEMOLOGICAL OBSTACLES IN HISTORICAL DEVELOPMENT IN CASE OF CONCEPT OF LIMIT:

It is useful to study the history of the concept to locate periods of slow development and the difficulties which arose which may indicate the presence epistemological obstacles. In the case of the history of the limit concept, we see that this notion was introduced to resolve three principal types of difficulty:

- Geometric problems (area calculations, consideration of the nature of geometric lengths, "exhaustion"),
- The problem of the sum and rate of convergence of a series,
- The problems of differentiation, (which come from the relationship between two quantities which simultaneously tend to zero).

There are four major epistemological obstacles in the history of the limit concept:

***a) The failure to link geometry with numbers:***

When the Greeks became interested in mathematics about 400 – 300 BC, we must ask why it happened that the limit concept was not clarified at the time. The problem of calculating the area of a circle, for example, supplied an opportunity to develop the tools very similar to the limit concept. Hippocrates of Chios(430BC) wanted to prove that the ratio between the areas of two circles is equal to the ratio of the squares of their diameters. He inscribed regular polygons within the circles and, by indefinitely increasing the number of sides, he approached the areas of the two circles. At each step the ratio of the area of the inscribed polygons is equal to the ratio of squares of the diameters, and it followed that, "in the limit", it would be true also for the areas of the circles.

This passage towards the limit, very sparingly explained, would be defined a year later, in terms of the method of exhaustion, credited to Eudoxus of Cnidos (408-255BC). The method is based on the principle of Eudoxus that "given two unequal lengths, if from the first is taken a part larger than its half, then from the remainder a part more than half what remains, and the process is repeated, then there will come a time when remains will be less than the second length". In other words, by successive halving we can attain a size as small as we wish. From this the principle of exhaustion follows which allows us to state that for any $\varepsilon>0$ there exist a regular polygon inscribed within a circle whose area differs from that of the circle by less than $\varepsilon$ . If the ratio of areas of two circles is $A_1/A_2$ and that of the squares of the radii is $r_1^2/r_2^2$, then we have one of these possible cases:

$$A_1/A_2<r_1^2/r_2^2,\ A_1/A_2 > r_1^2/r_2^2 \text{ or } A_1/A_2 = r_1^2/r_2^2$$

We eliminate the first two by the principle of exhaustion, and hence deduce the truth of the desired equality.

However, despite the fact that the exhaustion method seems extremely close to the notion of limit, we cannot affirm that the Greeks possessed the modern limit concept. The method of exhaustion is in essence a geometrical method of exhaustion is in essence a geometrical method which allows the proof of results without having to deal with the problem of infinity. It is applied to geometrical magnitudes, not to numbers. Each case is dealt with on an individual basis using a specific argument tailored to the geometrical context. There is no transfer from geometrical figures to a purely numerical interpretation, so the unifying concept of limit of numbers is absent. The geometrical interpretation, and its success in resolving pertinent problems, is therefore seen to cause an obstacle which prevents the passage to the notion of a numerical limit.

**b)The notion of the infinitely large and infinitely small:**

Throughout the history of the limit we meet the supposition of the existence of infinitesimally small quantities. Is it possible to have quantities which are so small as to be almost zero, and yet not having a specific ‘assignable’ size? Such philosophical problems have occupied the attention of numerous mathematicians who, like Newton, spoke of the “soul of departed quantities” at the time that they disappear to enable him to calculate their “ultimate ratio”. Euler freely used the notion of the infinitely small as a quantity that can, where appropriate, be considered equal to zero. D’ Alembert opposed the use of infinitely small quantities and sought to remove them from the differential calculus. He reasoned that a quantity is either something or nothing. If it is something it cannot be made zero and if it is nothing it is already zero. Thus the supposition that there is an intermediate state between the two he described as a wild dream. Cauchy also used the language of the infinitely small. In his Cours d’ analyse de l’Ecole Polytechnique of 1821, he defined a continuous function in these terms: *The function f(x) is continuous within given limits if between these limits an infinitely small increment i in the variable x produced always an infinitely small increment, f(x + i)-f(x), in the function itself.*

He explained the idea of an infinitesimal as follows:

*One says that a variable quantity becomes infinitely small when its numerical value decreases indefinitely in such a way as to converge to the limit zero.*

For Cauchy an infinitesimal is simple a viable which tends to zero. The idea of an ‘intermediate state’ between that which is nothing and that which is not, is frequently found in modern students. They often view the symbol ε as representing a number which is not zero yet is smaller than any positive real number. In the same way individuals may believe that 0.999 ….. is the ‘last number before 1’yet is not equal to one. There is a corresponding belief in the existence of an integer bigger than all the others, yet which is not infinite.

**c) Is the limit attained or not?**

This is a debate which has lasted throughout the history of the concept. For example, Robins (1697 – 1751) estimated that the limit can never be attained, just as regular polygons inscribed in a circle can never be equal to the circle. He asserted "We give the name ultimate magnitude to the limit which a variable quantity can approach as near as we would like, but to which it cannot be absolutely equal". On the other hand, Jurin (1685 – 1750), said that the "ultimate ratio between two quantities is the ratio reached at the instant when the quantities cancel out", "it is not a question whether the increment is zero, but that it is disappearing, or on the point of vanishing", "there is a last ratio of increments which vanish", "an increment born is an increment which starts to exist form nothing, or which begins to be generated, but which has yet to attain a magnitude that may be assigned to such a small quantity". D' Alembert insisted that a quantity should never become equal to its limit; "to speak properly, the limit never coincides, or never becomes equal to the quantity of which it is the limit, but is always approaching and can differ by as small a quantity as one desires".

The debate is still alive in our students. In a discussion one asked, "when n tends to zero, isn't equal to zero?" The following dialogue between students clearly illustrates the epistemological obstacle:

- The more n grows the more 1/n approaches zero.
- As much as one would like?
- No, because one day they will meet.

There are certainly many other obstacles to the notion of limit other than these three. The mistakes which students make are valuable indications for locating obstacles. The construction of pedagogical strategies for teaching students must then take such obstacles into account. It is not a question of avoiding them but, on the contrary, to lead the student to meet them and to overcome them, seeing the obstacles as constituent parts of the revised mathematical concepts which are to be acquired.

## [4] EPISTEMOLOGICIAL OBSTACLES IN MODERN MATHEMATICS:

It is interesting for mathematicians to look back at history and note the struggles that gave birth to modern ideas, leading to the logical state of the art today. However, the twentieth century quest for certainty based on a secure axiomatic foundation begun by Hilbert floundered on Gödel's incompleteness theorem, and so uncertainty remains. The introduction of Weierstrassian analysis, depending only on logical definitions of number concepts failed to eliminate the infinitesimal concepts that were an essential part of earlier mathematical culture. Although we may formulate definitions of limits and continuity in epsilon – delta terms, we still have occasion to use dynamic language of "variables tending to zero" in a manner analogous to that of Cauchy, with the resultant mental imagery linked to the "arbitrarily small".

Cognitively this phenomenon is to be expected. The idea of an “arbitrarily small” number is but the object produced by the encapsulation of the process of getting small in terms of *Dubinksy’s theory of encapsulation*. As Tall hypothesizes, the formation of a mental concept of an “arbitrarily small number” is a generic limit concept where the encapsulated object is believed to have the properties of the objects in the process. Thus the generic limit of a set of numbers which tend to zero is an arbitrarily small, yet non -zero, number. The concept is a natural consequence of the way in which the mind is hypothesized to work.

Hence, despite the attempts at banning infinitesimals from modern analysis, it continues to live in the minds and communications of professional mathematicians, even if it was eliminated from formal proofs. The return of the logically based infinitesimal in the work of Robinson (1966) re - opened the debate, which continues to be hotly contested. Although Robinson thought that his neat logical solution would solve the three hundred year conflict, this proved not to be so. For Robinson’s construction of a hyper real system containing real numbers and infinitesimals depends on a version of the axiom of choice and is there for non -constructive. This is becoming more a bone of contention as the arrival of computers begins to focus mathematicians on the pragmatic need to provide finite algorithms for constructive but the existence of a maximum value of a continuous function is not. The former asserts the existence of a zero of a continuous function between two places where the function has opposite signs and can be programmed on a computer by a simple bisection argument, but the later depends essentially on a non – constructive proof by contradiction.

In this way we see a recurrence of the problem of Lagrange as he attempted to remove the metaphysical ambiguity from the calculus: just as difficulty seems to be resolved, another seems to appear to take its place. This is typical of the complexity of the ideas in analysis and of the fundamental limit concept.

The limit concept is essential difficult may be seen in the way that it is defined in terms of an unencapsulated process: “give me an $\varepsilon > 0$, and I will find an N such that…..” rather than as a concept, in the form “there exists a function $N(\varepsilon)$ such that …” This means that the proof of the first theorem on the algebra of limits(that the sum, product etc of the limit of two sequences is the sum, product etc of the limit) is framed in process terms as the coordination of two process, rather than as the combination of two concepts. Were the latter to be the case, then the proof would follow a similar format, but it would have the advantage that it could be programmed on a computer in such a way that the proof of continuity is merely the operation of a computer algorithm. Yet this unencapsulated pinnacle of difficulty occurs at the very beginning of a course on limits presented to a naïve student.

## [5] THE DIDACTICAL TRANSMISSION OF EPISTEMOLOGICAL OBSTACLE

Given the complexities of modern mathematics and the cultural colourations in meaning, it is no surprise that such complex interactions affect students in their learning. In their human interactions they are very sensitive to tone of voice and implicit meanings and such ideas are conveyed to them by their teachers. Although such meanings may be avoided in written texts, they can passed on inadvertently from generation to generation as the teacher tries to "simplify" the complexity to "help" the students. When Orton (1980) investigated the limit concept in terms of a "staircase with treads", he showed a student the picture in figure -1 and asked

a) If this procedure is repeated indefinitely, what is the final result?
b) How many times will extra steps have to be placed before this "final result" is reached?
c) What is the area of the final shape in terms of "a", i.e. what is the area below the "final staircase"?

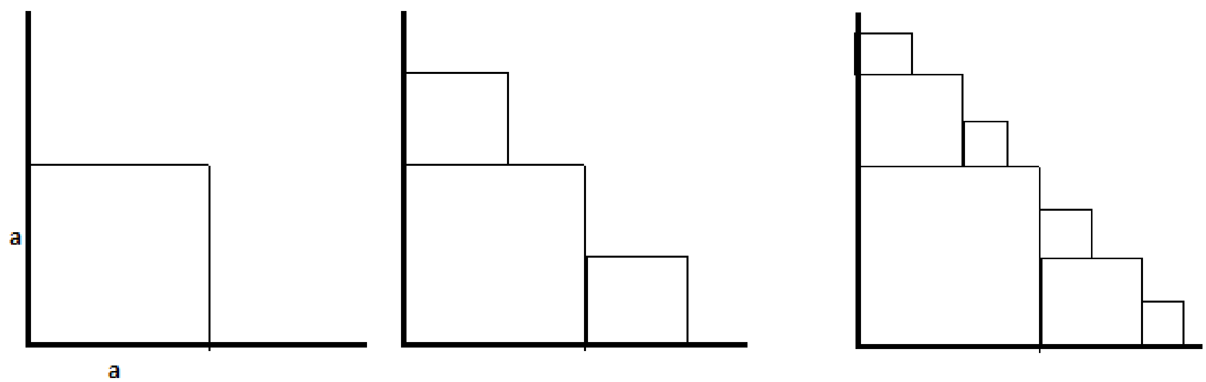


FIGURE 1

If a student gave a formula in response to (c) he asked:
Can you use this formula to obtain the 'final term' or limit of the sequence?
His justification for using this terminology way that:
*The expression "final term" was again used in an attempt to help the students understand the meaning of limits.*
However, in the light of what has been said here about generic limit concepts, it is evident that a phrase such as "the final staircase", far for helping the students with the formalities, is likely to create a generic limit concept in which the student imagines a staircase with an "infinite number of steps". This is precisely the response that it evoked.
In such ways, despite all our attempts to help students through the complexities, our attempts to "simplify" can lead directly to the cognitive obstacles which we have described earlier.

Such obstacles are almost certainly essential parts of the learning process. Davis & Vinner (1986) suggest that there are seemingly unavoidable stage in which misconceptions are bound to occur, in line with our assertion that such obstacles require a cognitive re -construction which are bound to involve a period of conflict and confusion. They too highlight the misconceptions that arise from the use of language evoking inappropriate images in spontaneous conceptions. Even though they attempted to teach a course in which the word "limit" was not used in the initial stages, they concluded that "avoiding appeals to such pre -mathematical mental representation fragments may very well be futile". They observed that another problem arises from the sheer complexity of the new ideas which cannot appear "instantaneously in complete and mature form" and so "some parts of the idea will get adequate representation before other parts". They give evidence, substantiating the discussion of Robert and Schwarzenberger, that specific examples dominate the learning, so that when monotonic sequence feature heavily in the student's experiences, it is not surprising that they dominate the student' concept images.

## Conclusion :

The diversity of conceptions, the richness and complexity of notions, and the cognitive obstacles makes the teaching of the limit concept extremely difficult. Numerous attempts have been made and the problem remains unresolved! On considering these attempts, it is possible to focus on certain fundamental points and to pose essential question.

In the first place, far many teachers seem to consider that it is sufficient to present a clear exposition of the limit concept to enable the students to understand. It is far more important that the students are made aware of the complexity of the notion and to reflect students own ideas and epistemological obstacles. Research so far shows clearly that the students own conceptions are quite varied, that they make fundamental mistakes and that they do not necessarily overcome epistemological obstacles. It is necessary to educate teachers to help them become aware of the problems involved. It is equally important for students to become explicitly aware of the essential difficulties. Experiment have been carried out in which their prior knowledge and understanding was mapped, which would necessarily be brought into play during the learning process. In particular, they were made aware of the different meanings of the words which they were going to use. This proved to be a valuable technique and enabled them to build on their own knowledge and understanding.

A further problem is that of the context in which the learning takes place. An effective apprenticeship needs to take place in a problem – solving context. The notion of limit has to be used to solve specific problems. It is therefore necessary to present situations in which the student can see that the limit is a useful tool, in which the limit is seen as part of the answer to questions which the student may have asked for him. This is often lacking in contemporary teaching. A definition of the notion of limit of limit is given, followed by a sequence of problems exercises, usually based solely on handling the algebra of the limit concept: the limit of a sum, of a product, of the composition of two series. We have already seen just how difficult the unencapsulated logical form of the limit definition is to handle for experts, let alone beginners.

It is important to consider the order in which the limit concepts are presented. Not only is there the question of designing a logical mathematical order of concepts, but also the cognitive appropriates of the curriculum sequence and of the problems to be solved. It is now well -established that in the transition to advanced mathematical thinking a purely logical sequence of topics, in which the mathematical concepts are introduced through definitions and logical deductions, is likely to be insufficient.

**REFERENCES :**


1. *Bell, E. T. (1945). Development of mathematics. New York: McGraw-Hill.*
2. *Kennedy, J., & Ragan, E. (1989). Function. In J. Baumgart, D. Deal, B. Vogeli, & A.Hallerberg (Eds.), Historical topics for the mathematics classroom (pp. 312-313).Reston, VA: National Council of Teachers of Mathematics.*
3. Sierpinska, Anna.(1992).“On Understanding the Notion of Function.” In *The Concept of Function: Aspects of Epistemology and Pedagogy*, edited by Guershon Harel and Ed Dubinsky, pp. 25–58. Washington D.C. : Mathematical Association of America.
4. *Siu, M. K. (1995). Concept of function – its history and teaching. In. F. Swetz, J. Fauvel,O. Bekken, B. Johansson, & V. Katz (Eds.), Learn from the masters (pp. 105-121). Washington, DC: Mathematical Association of America.*
5. Crombie, A. C. (1959). *Medieval and early modern science, Volume1. Science in the middle ages: V-XII centuries.* Garden City, NY: Doubleday.
6. Kleiner, I. (1989). Evolution of the function concept: A brief survey. The College Mathematics Journal, 20, 282-300.
7. *Katz, V. J. (1998). A history of mathematics. An introduction (2nd ed.). New York:Addison-Wesley.*
8. D. O. Tall: Conflicts and Catastrophes in the Learning of Mathematics,*Mathematical Education for Teaching*, 2, 1977.
9. D. O. Tall: A Long-Term Learning Schema for Calculus and Analysis *Mathematical Education for Teaching*, 2, 1973.

The International Journal of Analytical and Experimental Modal analysis

An UGC-CARE Approved Group - II Journal

An ISO : 7021 - 2008 Certified Journal

ISSN NO: 0886-9367 / web : http://ijaema.com / e-mail: submitijaema@gmail.com

IJAEMA

# Certificate of Publication

This is to certify that the paper entitled CERTIFICATE ID:IJAEMA/4530

**"OBSTRECLES WHICH STAND IN THE WAY OF STUDENTS LEARNING THE LIMIT CONCEPT"**

Authored by :

Dr. Dheeraj Kumar

From

Patna University, Patna, Bihar

Has been published in

IJAEMA JOURNAL, VOLUME XII, ISSUE IX, SEPTEMBER- 2020

ACADEMIC EXCELLENCE

UGC APPROVED

Michal A. Olszewski Editor-In-Chief

IJAEMA JOURNAL

6.3 IMPACT FACTOR

ISO International Organization for Standardization 7021-2008

http://ijaema.com/