ASSESSMENT AND EVALUATION USING COMPREHENSION TEST IN CALCULUS

Dr. Dheeraj Kumar[1], Dr. Vinay Kumar Kantha[2]

[1]*PG Department of Mathematics, Patna University, Patna, Bihar.*
Email: dheerajkr2011@gmail.com
[2]*Rtd. Associate Professor, Department of Mathematics, B.N.College,Patna University, Patna, Bihar.*

Abstract

Conceptual learning should be the main aim of Mathematics as a course. Implementations for improving reasoning skills of the students are extremely important. However, mathematics may sometimes happen to become a routine operation. Assessment and Evaluation methodology adopted by the teachers, in particular, becomes the basis of Student's in determining the learning method. If the said test does not assess conceptual knowledge, then the students will remain to be inclined to learning by rote. So even in a case where student is accurately responding to the tests, it is rather difficult reveal her/his concept knowledge i.e. to identify that actually how much depth of knowledge the student have and what concept image She/He actually have. Use of comprehension tests will prevent rote learning and help identify/rectify the faulty concept images of the students.

***Keywords:* Comprehension test in calculus, definition of functions, image concept, limit, derivative.**

1. INTRODUCTION

Calculus is the mathematical study of change, in the same way that geometry is the study of shape and algebra is the study of operations, their application and structures. However, the concept of limit underlies the idea of change in calculus, unlike arithmetic or classical algebra or geometry. Sometimes calculus is explained just a shorthand for "differential calculus" and "integral calculus". So it is mostly "calculus" when one is concerned with finding derivatives and integrals, and solving (easy) differential equations, with emphasis on the mechanical part. Analysis is a much broader term, that includes the concepts and proofs, concerning calculus (continuity, differentiation, integration), but many others, including for example Measure Theory and Functional Analysis.

Recently Tall (2004, 2006) proposed a categorisation of cognitive growth into three distinct but interacting developments which he calls "conceptual-embodied world" growing out of our perceptions of the world; "the proceptual –symbolic world" of symbols which we use for calculation and manipulation; and "the formal – axiomatic world" based on properties expressed in terms of formal definitions that are used as axioms to specify mathematical structures.

Researchers such as Vinner (1989) claim that students tend to use algebraic representation and methods when solving calculus question, avoiding the visual methods that would be expected to be associated with the conceptual – embodied world; however, research by Tall and Watson (2001) suggests this may be an artefact of the teaching and assessment the students have received, and the way they were encouraged to construct their knowledge. In their study of the manner in which students build up meaning to sketch the gradient graph of a given function, one teacher privileged 'a visual –enactive approach ' whereby she followed the shape of the graphs in the air with her hand encouraging her students to follow her lead – thus building a physical sense of the changing gradient visual and symbolic ideas were deliberately linked by the teacher. The students of this teacher outperformed the other students in the study who were taught by others teachers using a more traditional approach to graph sketching and development of gradient of these graphs.

The concept image is the mental pictures that students construct for each mathematical concept. The concept definition is a formal definition of a mathematical concept (Lambertus 2007[1]). The distinction between concept image and concept definition arose originally in the work of Vinner, Tall, and Dreyfus. In their usage, a concept definition is a customary or conventional linguistic formulation that demarcates the boundaries of a word's or phrase's application. On the other hand, a concept image comprises the visual representations, mental pictures, experiences and impressions evoked by the concept name (Thompson 1994[2]). Concept definition refers to the mathematical definition of a concept, while concept image is everything associated in somebody's mind with the concept name. The students' prior exposure to the concepts is detrimental due to the inappropriate existing network. As Ausubel stated, "The most important single factor influencing learning is what the learner already knows" (DeMarois 1996). The concepts should be effectively used at right place and at right time. Van de Wella defined mathematicaly.

Students need to understand conceptual knowledge of mathematics
ii. Students need to understand procedural knowledge of mathematics
iii. Students need to understand relationships between conceptual and procedural knowledge

In mathematics, procedural knowledge defines symbols, rules and knowledge used in solving mathematical problems. On the other hand, conceptual knowledge is described as mathematical concepts and relationship to each other (Baykul, 1999). However, it is not possible to separate conceptual knowledge and procedural knowledge precisely. If a person has conceptual knowledge which constitutes procedural knowledge, one can make strong connections between basic concepts, can reach to solutions by using data given, find mathematical construction wanted, and can easily explain mathematical construction relating with rules and symbols which one knew by ones conceptual knowledge. In mathematics, permanent and functional learning can be possible only by balancing procedural and conceptual knowledge (Noss and Baki,1998[3]).It is hard for a student to acquire a correct concept definition when conceptual learning does not occur. As a result, the student forms his/her own concept image. One cannot expect an exact overlapping of the concept image formed by the student with the concept definition. In an ideal learning, efforts should be directed toward forming a mathematically correct concept image. It is also important to reveal students' concept images. For incorrect concept images constitute significant obstacles to students' accurate learning. Therefore, a teacher's assessment instrument can reveal students' concept images. Conradie and Frith (2000[4]) developed a"comprehension test", which mainly aims to reveal students' concept images. In the present study, by taking Conradie and Frith's (2000) test as a reference, a "comprehension test" has been developed which involves the basic concepts in Calculus such as limit, continuity, and derivative.

2. COMPREHENSION TEST: Read the following definitions and answer the following questions.

Definition 1: By a δ-neighborhood of a point 'a' we mean the set of all points $|x - a| < \delta$ such that , where δ>0

Definition 2: Let $A \subset R$ and let 'a' be a point which is not necessarily in A (x ε R)We call 'a' an accumulation point of A if in each δ -neighborhood of 'a' there is at least one point which is in A and distinct from 'a'

Definition 3: Given subset A of R , continuity of a function f:A→R at a ε A means that for every ε>0 there exists a δ(ε,a)>0 such that for all x ε A:

$|x - a| < \delta \Rightarrow |f(x) - a| < \varepsilon$

If 'a'*is not an* accumulation point of A, is assumed continuity at c.

Question: Let f be a function that f{1,2,3}→R,f(x)=x^2 . The graph of this function is given below.

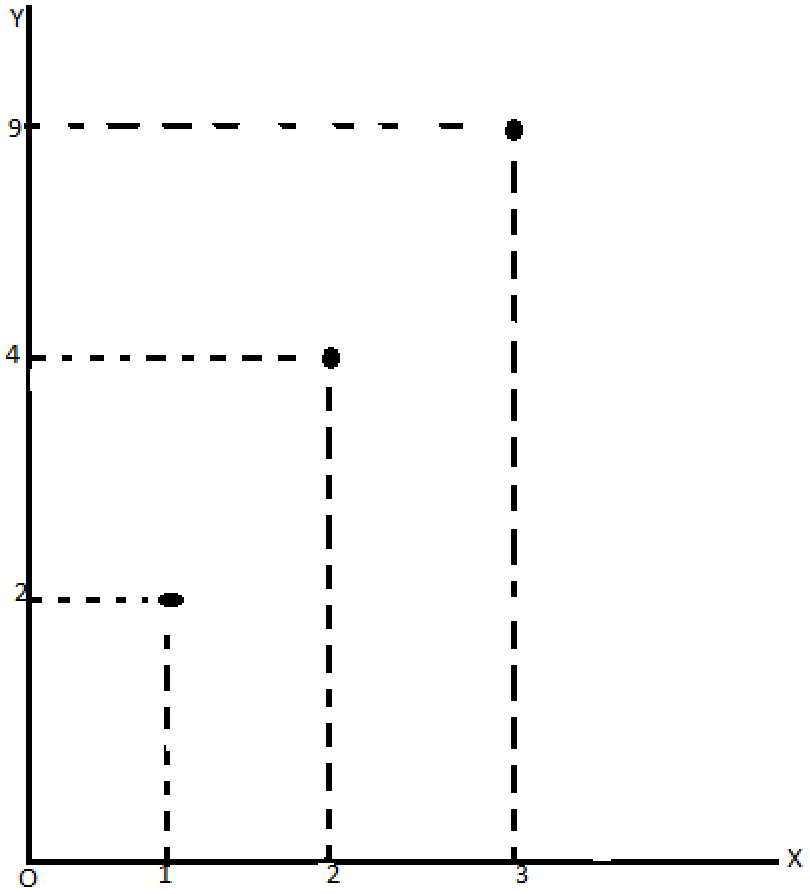


Fig.1: Graph of function.

a) Is this function continuous at the point x=2?
b) “if functions f is continuous, its graph can be drawn without removing your pen from the paper” the statement is true or false?
c) For a function f to be continuous at a point, should there be a limit to that point?
Definition 1: Let f be a function defined in the open interval (a,b) and
c ε (a,b), the limit

$$\lim_{h\to 0}\frac{f(\mathrm{c}+\mathrm{h})-\mathrm{f(c)}}{h}$$

exists, then function *f* is said to be differentiated at point cand is denoted by

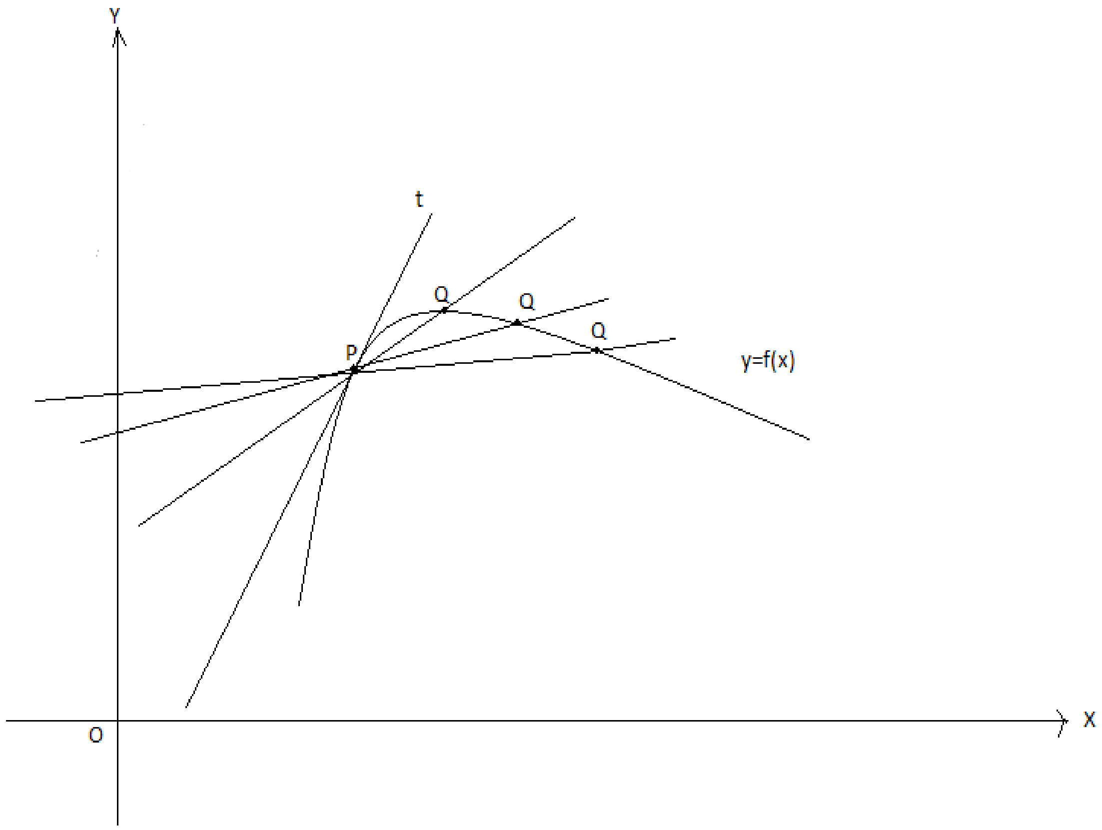
$$f'(c)=\lim_{h\to 0}\frac{f(\mathrm{c}+\mathrm{h})-\mathrm{f(c)}}{h}$$

Definition 2: The tangent line to $y = f(x)$ at(c,f(c))is the line through (c,f(c))whose slope is equal to f’(c) the derivative of f at c.
Definition 3: If y=f(x) be a curve and we want to find the tangent line to curve at the point *P(a,f(a))* then we consider a nearby point $Q(x, f(x))$, where , and compute the slope of the

secant line PQ :

$m_{PQ}= \frac{f(x)-f(a)}{x-a}$

Then we let Q approach P along the curve by letting x approach a. If m_{PQ}approach m then we define the tangent t to be the line through P with slope m.



Fig,2: The graph of tangent line

- *Question:* As follows the graph of $f(x)$ = Isin*x*l.Show that the following questions by examining the graph of this function and the above definitions

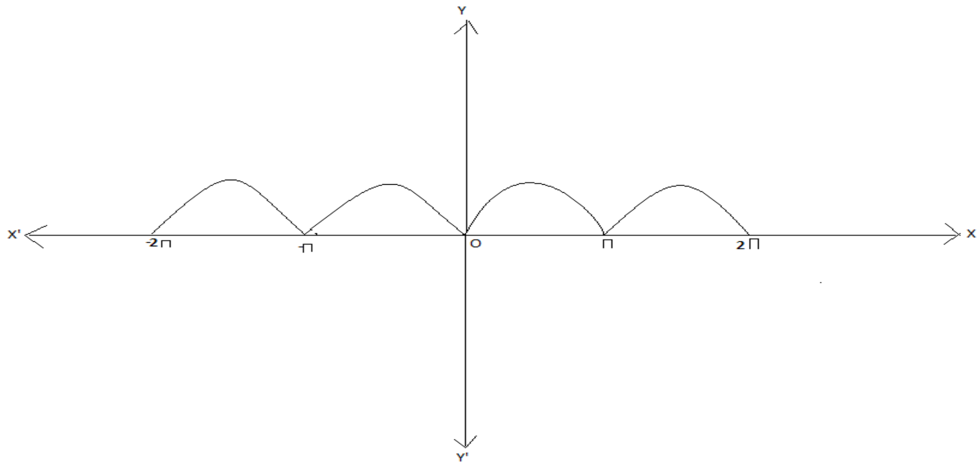


Fig.3: The graph of function

a) How many tangents can be drawn at the point $x=\pi$ to this function?
b) Is there a derivative of the function f(x) at the point $x=\pi$?

Definition 1: : Let $A \subset R$ and let 'c' be a point which is not necessarily in A (x ε R)We call 'a' an accumulation point of A if in each δ -neighborhood of 'a' there is at least one point which is in A and distinct from 'c'

Definition 2: Let $f : A \rightarrow R$ be a function and c accumulation point of $A \subset R$ Then the formula$\lim_{x\to c} f(x) = L$
means for each real ε >0 there exists a real δ(ε)> 0 such that for all x with 0 <|x - c|< δ, we have
$|f(x) - L| < \varepsilon$.
Question: $\lim_{x\to -3} \sqrt{x}$
a) Show that whether $\lim_{x\to -3} \sqrt{x}$is can be calculated or not?
b) To this question, what is A set of given limit definition in the above?

3. BENEFITS OF COMPREHENSION TEST

a) Mathematics is a way of thinking (Pesen and Odabas 2000). Then, thought should be brought into prominence. Question types such as "prove the x theorem" or "state and prove y theorem" lead students to verbatim memorizing (Conradie and Frith 2000). Comprehension tests are required to improve reasoning skill.
b) It is important in identifying students' concept images. It is difficult to identify students' concept images by using question types such as "Calculate the limit" or "Is function of continuous at point x=a?".
c) Since this type of test question the content of any subject step by step, it is easy to identify what deficiencies or difficulties the students have at any step regarding the subject.

3.1. ANALYSIS OF SOME PROBLEMS OF CALCULUS

For the purpose of analyzing some practical & real examples, we will hereby take some examples from questions asked in 2012, 2013, & 2014 in CBSE 10 +2 Examinations.

a. For problem No.11 of year 2014

Students have to find gof(x) & fog(x) we can see

$$gof(x) = g\{f(x)\}$$

$$= g(x^2+2) \qquad \{\text{fitting the value of } f(x)\}$$

$$= g(y) \qquad \{\text{let } y=x^2+2\}$$

$$=\frac{y}{y-1}$$

$$=\frac{x^2+2}{x^2+2-1}$$

$$=\frac{x^2+2}{x^2+1}$$

It is easily see that process of fitting the values for definition of given function gives result. But if cannot judge the conceptual part of topic i.e, for what value of x gof is defined. Is domain of gof is equal to domain of f(x) or g(x).

Now, $fog(x) = f\{g(x)\}$

$$=f\left(\frac{x}{x-1}\right)$$

$$=\left(\frac{x}{x-1}\right)^2+2$$

$$= \frac{x^2+2(x-1)^2}{(x-1)2}$$

$$= \frac{3x^2-4x+2}{(x-1)^2}$$

Clearly fog(x) is not defined for x = 1.But gof(x) is defined for all value of x.

Hence we see domain of fog(x) is not equal to domain of gof (x). Here question can be asked in different manner such as after finding fog & gof represents them diagrammatically.

b. For question no.11 of CBSE 2012 for topic of function we can see

For one-one let $x_1, x_2 \in A$ such that $x_1 \neq x_2$

then $x_1 - 2 \neq x_2 - 2$ …………(1) & $x_1 - 3 \neq x_2 - 3$……………..(2)

divide (1) by (2)

$\frac{x_1-2}{x_1-3} \neq \frac{x_2-2}{x_2-3}$

$f(x_1) \neq f(x_2)$

hence f is one- one

For onto let y= f(x)

then $y=\frac{x-2}{x-3} \Rightarrow$ $yx - 3y = x - 2$

$\Rightarrow$ $yx - x = 3y - 2$

$\Rightarrow$ $x(y-1) = 3y - 2$

$\Rightarrow$ $x = \frac{3y-2}{y-1}$ hence $\forall\ y \in R-\{1\}\ \exists\ x \in A$ therefore f is onto.

Here we can see this question only Evaluate the procedure knowledge of one –one, onto & invertivility of the function, if this question has been asked in the way

Let A & B be two sets and f is a function from A to B such that $f(x) = \frac{x-2}{x-3}$ then what should be the set A & B for which f is one – one, onto function and it posses its inverse.

Similarly for question no.11 of function for year 2013 it could be asked in this way consider $f: A \to B$ given by $f(x) = x^2 + 4$ then what should be A & B for f is invertible.

Similarly for topic of differentiation also we also see question no.14 of 2014, question no.14,15,16(OR part)of set-1 2013 question no.14,of 2012 set-1 of CBSE all questions only examine the procedure knowledge i.e mechanical knowledge of students .Testing of conceptual understandings are missing in these questions . If these questions had been asked in another way, then students understanding could be better evaluated. For example question no.14 of 2014 CBSE board paper, question could be asked in the following manner

Find $\frac{dy}{dx}$ for the functions at $\theta = \frac{\pi}{4}$

$$X = ae^{\theta}(\sin\theta - \cos\theta)$$

$y = ae^{\theta}(\sin\theta + \cos\theta)$ and also find the values of θ for which $\frac{dy}{dx}$ does not exists and why?

For testing of conceptual understanding of differentiation following types of questions may be asked:

1. If f(x) = $\begin{cases} x+1 & for \quad x<1 \\ 1 & for \quad 0 \leq x \leq 1 \\ 2x^2+4x+5 & for \quad x>1 \end{cases}$

   find f'(x) for all values of x for which it exists. Does $\lim_{x \to 0} f'(x)$ exists?

2. Differentiatethe following function
   $\log_{v(x)} u(x)$ {that is the logarithm of u(x) to the base v(x);v(x)>0

3. What conditions must the co-efficient α,β,a,b,c satisfy in order that $\frac{\alpha x+\beta}{\sqrt{ax^2+2bx+c}}$ shall everywhere have finite derivative that is never zero.

4. CONCLUSION

It is widely believed that mathematics is based on abstract thinking. At times, students' attempts at memorizing can be attributed to the incorrect pedagogical attitude of teachers. Such incorrect pedagogical attitudes may also include assessment tests. Repetition of the same question types in assessment instruments may lead students to perform without thinking. For instance, Tall and Vinner (1981) and Ozmantar and Yesildere (2008) demonstrated in their studies that the students had the concept image that "the graph should be one piece or one should be able to draw the graph in one move without raising the pencil from the paper" for a function to be continuous. A mathematics instruction based on memorizing is undesirable in which thinking is of secondary importance.

We believe that using "Comprehension test" will be useful in identifying concept images. By identifying students' concept images, their pedagogical misconceptions and conceptual difficulties will be prevented at the least.

Thus, even simply identifying concept images is extremely important.In a test content with step-by-step questioning, it is easy to identify the steps in which students have misconceptions. This way, concept images can be easily converted into concept definitions through feedback.

REFERENCES

1. *Lambertus, A., J. (2007). Students' Understanding of the Function Concept: Concept Images and Concept Definitions, North Carolina State University (Unpublished Master Theses).*
2. *Thompson, P., W., 1994. Students, functions, and undergraduate curriculum, In E. Dubinsky, A.H. Schoenfeld, and J. J. Kaput (Eds.), ResearchIn Collegiate Mathematics Education, 1 (Issues In Mathematics Education Vol. 4, p.21-44). Providence, RI: American Mathematical Society.*
3. *Noss, R., & Baki, A. (1996). Liberating school mathematics from procedural view, Journal of Hacettepe Education, 12, 179-182.*
4. *Conradie, J., & Frith, J. (2000). Comprehension test in mathematics, Educational Studies in Mathematics, 42, 225-235*

5. *John von Neumann, J., "The Mathematician", in Heywood, R. B., ed., The Works of the Mind, University of Chicago Press, 1947, pp. 180–196. Reprinted in Bródy, F., Vámos, T., eds., The Neumann Compendium, World Scientific Publishing Co. Pte. Ltd., 1995, pp. 618–626.*
6. *Tall, D. (1996). Functions and calculus. In A. J. Bishop, M. A. Clements, C. Keitel,J. Kilpatrick & C. Laborde (Ed.), International handbook of mathematicseducation (pp. 289-325). Dordrecht: Kluwer Academic Publishers.*
7. *Tall, D. O. (2004). The three worlds of mathematics. For the Learning of Mathematics, 23 (3). 29–33.*
8. *Gray, E. M. & Tall, D. O. (1994). Duality, ambiguity and flexibility: A proceptual view of simple arithmetic, Journal for Research in Mathematics Education, 25 2,115–141.*
9. *Vinner, S.: 1989, 'The avoidance of visual considerations in calculus students', Focus on Learning Problems in Mathematics 11(2), 149-156.*
10. *Victor J. Katz (1995), "Ideas of Calculus in Islam and India", Mathematics Magazine 68 (3): 163-174 [165-9 & 173-4]*
11. *Broadbent, T. A. A. (October 1968), "Reviewed work(s): The History of Ancient Indian Mathematics by C. N. Srinivasiengar", The Mathematical Gazette* ***52*** *(381): 307–8*

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE

Journal of Information and Computational Science

UGC - Care Group - II Certified Journal

ISSN NO: 1548-7741 / web : www.joics.org / E-mail : submitjoics@gmail.com

Certificate of Publication

This is to certify that the paper entitled

ASSESSMENT AND EVALUATION USING COMPREHENSION TEST IN CALCULUS

Authored by :

DR. DHEERAJ KUMAR

From

PG Department of Mathematics, Patna University, Patna, Bihar.

Has been published in

JOURNAL OF INFORMATION AND COMPUTATIONAL SCIENCE,VOLUME 10, ISSUE 8, AUGUST-2020

ज्ञान-विज्ञान विमुक्तये

An Emblem Symbolising The Future

S. Joseph

Joseph Sung
Editor-In-Chief
JOICS

6.2 IMPACT FACTOR

ISO International Organization for Standardization 7021-2008